A Conserving Optimal Least-Squares Finite Element Method for CFD Problems

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ABSTRACT

In the past decade, advances in Least-Squares Finite Element Method (LSFEM) in incompressible flow, especially in the vorticity-velocity-pressure formulation, have been applied to many industrial applications due to their numerical efficiency, robustness, and guarantee of unique solution [1]. Along with the method's wider applications, some obvious mass conservation violations have been observed [2, 3]. Applying a weighting to the functional or applying a Lagrangian multiplier to the mass conservation equation has proven to be effective, but these methods are either effective for only a subset of problems or lead to the loss of the positive definiteness advantage of the LSFEM.

This paper shows that the mass conservation error in the LSFEM solution can be found from a residual function normal to the desired divergence free velocity field based on the incompressibility constraint. The error residual functional, as shown in a Poisson equation form, can be solved directly with natural Neumann boundary constraint. The pressure correction term can also be found by substituting the corrected velocity field into the momentum equations, either numerically or semi-analytically, in the iterative solution process. Since the culprit of the continuity violation of the LSFEM method is due to the relative magnitude sensitivity of the numerical minimization process of the least-squares constraint equations, the method augments the existing LSFEM and ensures the solution to be conserving, in an integral sense, at each solution step without any weighting or additional constraint.

Numerical benchmarks and results are presented with comparison to published results. Several possible alternative refinements are also discussed.

REFERENCES

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