ADVANCES IN HYPERELASTIC FINITE ELEMENT MODELING OF BIOLOGICAL TISSUES: EXPLICIT STRAIN ENERGY FUNCTION SPECIFICATION

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INTRODUCTION

Finite element modeling (FEM) is a suitable method to study the mechanics of biological tissues, but its utility for studying soft tissues has been limited by problems with numerical stability at large, complex deformations. Here, we describe a new paradigm for finite element tissue modeling that greatly increases numerical stability: explicit specification of a strain energy density function using Sefea™ (Strain-Enriched FEA, AMPS Technologies). This method relies on numerical differentiation of the strain energy function to calculate the stress and tangent tensors for each integration point (instead of using complex analytical expressions). We create an image-based model of a hamstring musculotendon unit—the biceps femoris longhead—to demonstrate this method's utility and stability.

METHODS

Typically, FEM of hyperelastic materials relies on a strain energy density function. For example, a simple isotropic, quasi-incompressible hyperelastic material model is a Mooney-Rivlin solid, whose strain energy function is given by the expression

$$W = C_{10}(\tilde{I}_1 - 3) + C_{01}(\tilde{I}_2 - 3) + W(J)$$

Where W is the strain energy (a scalar), C_{10} and C_{01} are material constants, $\tilde{I}_1$ and $\tilde{I}_2$ are scalar deviatoric invariants that are functions of the deformation tensor, and $W(J)$ is a volumetric strain energy term of the relative volume J in order to enforce a nearly-incompressible material response. More complex strain energy functions can be used for transversely isotropic biological tissues. Most solvers require calculation of the stress and tangent tensors that are functions of the derivatives of the strain energy density with respect to the right Cauchy-green deformation tensor C:

$$\sigma = f\left(\frac{\partial W}{\partial C}\right)$$
$$\mathbb{C} = f\left(\frac{\partial^2 W}{\partial C^2}\right)$$

where σ is the Cauchy stress and $\mathbb{C}$ is the tangent tensor. These tensors are typically calculated analytically, but Sefea™ calculates them with numerical differentiation of the strain energy density function.

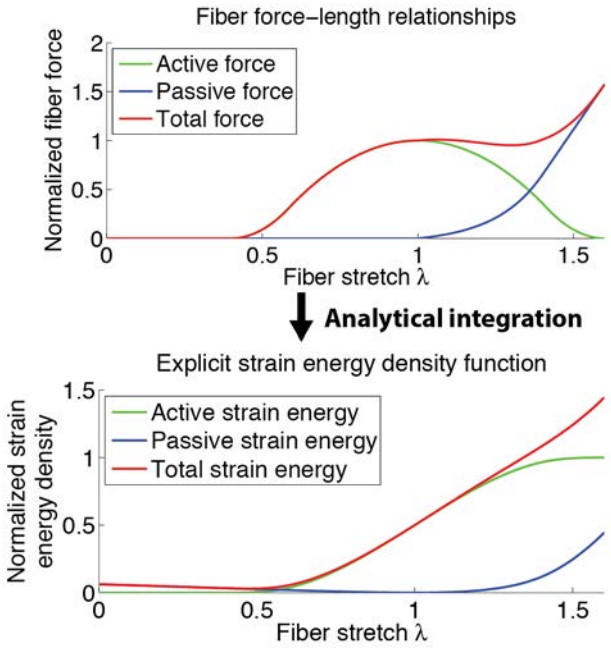

Figure 1: We integrate the force-length relationship of muscle [1] to obtain an explicit strain energy function for muscle.

The advantages of Sefea™ include i) using enhanced 4-node tetrahedral elements to enable automatic meshing and ii) specifying an explicit strain energy density function rather than the stress and tangent tensors. We determine the along-fiber strain energy function of muscle by integrating the force-length curve of muscle [1] (Figure 1). The active force curve scales with muscle activation. The same operation is performed to determine the

tendon strain energy function. We used material parameters for muscle and tendon from a previous study [1].

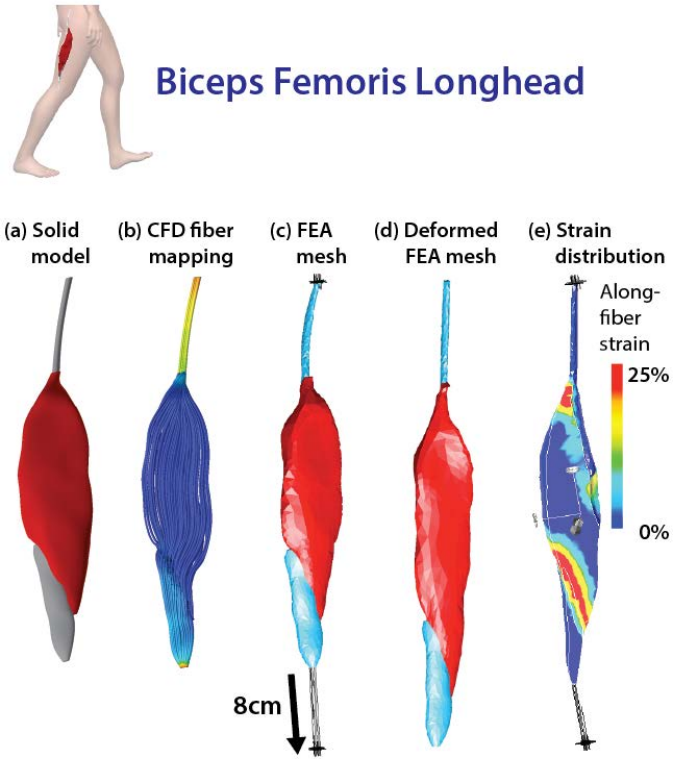

Figure 2: (a) MRI-based solid model of the biceps femoris longhead [2]. (b) Computational fluid dynamics determines the fiber directions for the transversely isotropic formulation [3]. (c) The solid was meshed automatically with enhanced 4-node tetrahedral elements and the distal tendon was displaced by 8cm. (d) The deformed mesh reaches full convergence. (e) Strain distributions within the muscle are non-uniform.

We created a 3D model of the biceps femoris longhead based on MRI data [2] (Figure 2a). We used used computational fluid dynamics to specify fascicle trajectories for the transversely isotropic material models [3] (Figure 2b). We used automatic tetrahedral meshing and simulated a long eccentric contraction by ramping the muscle activation from 0 to 1 while displacing the distal tendon by 8cm (well above the stretch during sprinting of 5cm [4]) (Figure 2c).

RESULTS

The model was numerically stable using Sefea^{TM}, reaching full convergence (Figure 2d). The deformed model at a physiological 2cm displacement (approximately 40° of knee extension [2]) shows non-uniform strain distributions (Figure 2e). Non-uniform strain distributions have been shown characteristically in previous finite element studies of eccentric contraction [1,2,4].

DISCUSSION

Our new finite element modeling paradigm of explicit strain energy function specification within Sefea^{TM} significantly enhances numerical stability. Furthermore, it enables fully automatic meshing of biological tissues of arbitrary shape and complexity using 4-node tetrahedral elements. These elements are also stable in contact scenarios—typically not the case for non-linear elements. This new paradigm and technology will empower hyperelastic finite element studies of biological tissues that would be otherwise unfeasible.

REFERENCES

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